Geometry Problems and Projects: Triangles

for use with The Geometric Supposer

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Geometry Problems and Projects: Triangles

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Richard Houde teaches mathematics and chairs the Mathematics Department, grades 6-12, in the Weston Public Schools, Weston, Massachusetts. He was the first teacher to integrate *The Supposer* formally and fully into geometry instruction. He has worked closedly with Yerushalmy and EDC in thinking about how to integrate inductive reasoning into geometry learning and teaching. He has spoken widely and presented a range of workshops for elementary and secondary teachers on using *The Supposer* in the classroom. Houde holds a B.S. Ed. from Bridgewater State Teachers College, an M.A. in mathematics from Clark University, and an Ed.D. in mathematics education from the University of Tennessee.

The **Center for Learning Technology** was established in 1982 by Education Development Center, which for over twenty-five years has pioneered the use of new technologies as tools for teachers and learners. In addition to software development for language arts and mathematics, the Center's activities include research, curriculum development, policy analysis, and videotape and videodisc production. The Center is a member of the Harvard-based U.S. Department of Education/OERI funded Educational Technology Center.

About *The Supposers:*

The Geometric Supposers are a series of microcomputer programs, each of which deals with a broad section of the geometry curriculum. Titles in the series are: *The Geometric Supposer: Triangles, The Geometric Supposer: Quadrilaterals,* and *The Geometric Supposer: Circles* for use primarily at the secondary level; and *The Geometric preSupposer* for use in elementary and middle schools. *The Supposer* series is published by Sunburst Communications, Inc. and was designed by Judah L. Schwartz, Michal Yerushalmy, and the Center for Learning Technology, Education Development Center.

Table of Contents

Problems and Projects: Triangles

Introduction

This is the first in a series of *Problems and Projects* books designed for use with *The Geometric Supposer*. These problems and projects, dealing primarily with triangles, have been selected from the large number of activities that we and other teachers have developed and used successfully in classrooms over the last three years.

As a group, these problems and projects represent:

- **the full range of topics covered** in the triangles portion of the typical geometry curriculum *and then some*;
- **a variety of problem types and formats** which take advantage of the SUPPOSER's features; and
- **a range of difficulty and complexity**.

Problems are an essential component in any instructional process. They focus attention and energy and guide students in the application, integration, and extension of knowledge. This is particularly so in an open-ended software environment such as *The Supposer. The Supposer* is a tool with a set of capabilities; it has no explicit instructional framework or imperative. The utility and power of *The Supposer* become apparent only in the context of a task or a problem.

We hope that these problems will help you tap the power of *The Supposer.* In addition, we hope that you will view this book as a source of raw material to be refined and adapted to meet your students' needs and as a point of departure for developing your own exercises and activities.

Michal Yerushalmy
Richard Houde

Index by Subject

About Geometry Problems and Projects: Triangles

The Geometric Supposer is an open-ended, flexible tool that can be adapted to meet a range of learning and teaching needs and objectives. The same holds true for the problems and projects in this book. They provide an opportunity to investigate or to reinforce geometric concepts and relationships. They can be modified for use in different instructional settings, by teachers with a wide variety of styles, and with students of all ability levels.

This section provides some general information about the problems and then offers some suggestions for adapting and modifying problems to meet your needs. We suggest that you read this section, try some problems on your own, and use them with your students. Then come back to this section to think about these issues in terms of your own experience.

General Information

Format

The Student Problem sheets are reproducible blackline masters. The format of these problems was designed to ensure a clear presentation and to suggest a style of work. Each Student Problem sheet starts with a definition of the **task** so that from the outset the student has a sense of the objective. It is followed by the **procedure** which provides directions for constructions, measurements, conjectures, etc. A drawing of the figure/construction to be explored provides additional information. Some problems include charts and tables to guide student inquiry. More frequently, problems provide an open **drawings and data** space so that students can record figures and measurements. Students complete the activity by making **conjectures**. Lines are provided for students to write their observations, definitions, construction procedures, or conclusions.

The Teacher Notes page for each problem includes a **reduced student problem sheet** and **notes** about conjectures, proofs, and use. The **comments** space enables you to record your students' reactions and ideas for modifying the activities.

Order/Index

Problems are numbered sequentially, but they need not be used in that order. When problems have more than one part (e.g., T 40 Part A, T 40 Part B, T 40 Part C), each part begins on a new page to maximize flexibility of use in the classroom. The order of problems in the book is roughly as follows: it starts with triangles, moves on to geometric constructions, and then to relationships among elements and triangles. These are not hard and fast divisions. The Index by Subject on page 2 can help you identify or select problems by topic in the geometry curriculum.

Proof

These problems are designed primarily to help students explore geometric ideas. They emphasize inductive reasoning: looking at particular shapes and constructions; gathering, organizing and analyzing data; developing and testing working hypotheses; and making conjectures and generalizations. Some problems call for students to provide supporting arguments (informal proofs) with their conjectures. You can, of course, add a requirement of proof to many of the problems. Some teachers ask students to produce conjectures for a problem, and then later in the year, return to the same problem, asking students to prove their conjectures.

Time/Difficulty

There are no simple answers to questions such as: How long should it take students to work on a particular problem? Is this a simple or difficult problem? Therefore, a wide variety of problems and projects have been included. The best way to assess the time requirement and difficulty level for your students is to work the problems yourself before presenting them to the class. By working the problems, you will be able to determine the time and the mathematical thinking required and better understand their potential. Working the problems will also help you see how they fit with your curriculum objectives and your students and help you use them most effectively.

The decision to assign a given problem as lab work or as homework must take into consideration the amount of time your students take to work through a problem, student access to computers, and your flexibility in scheduling time in the computer lab. Some teachers have students use labtime for investigating a problem, developing initial conjectures, and collecting data, and then use homework time for writing conjectures formally and for devising proofs.

Some of these problems were designed as "projects." These might be considered the geometry course equivalent of a research or term paper. Projects tend to be multi-staged, or more complex, or open-ended, or all of the above. They can be used as summary activities or independent research projects, or be broken down into smaller segments. Again, it depends on your style, your objectives, and your students. For example problems T 72, T 73, and T 74 might constitute a project.

You may also find that some of the problems are too simple or segmented. In that case, you may choose to group some problems together or to combine several into a project.

Labels/Orientation

Many of the problems include labeled drawings of shapes. In watching students work with *The Supposer*, especially those who are new to the program, we have observed that drawings in a textbook or on a problem sheet can be the source of some confusion. *The Supposer* generates random shapes displayed in random orientation. When students try to recreate a triangle from a drawing, they may discover differences in the labeling of the vertices and the orientation of the triangle. While initially this may be confusing, it also provides an opportunity to discuss the meaning of labels and whether the orientation of a triangle alters its properties.

Making These Problems Work For You

In watching different teachers and students work, we have noticed great variation in the amount of structure that teachers provide and students require to work effectively and productively on a problem. In writing these problems and in designing the format, we have taken what we consider to be a middle-of-the-road approach. With every problem, there are choices to be made that will determine the degree to which learning is structured.

Here are some ways in which problem structure can be manipulated:

Defining the Task
The statement of the task can vary in the amount of direction it provides.

- A problem can be a totally open-ended affair such as "Investigate altitudes."
- A problem can offer a set of instructions that move you from sub-task to sub-task. (This is the style of most of the problems in this book.) For example,
  a) Draw an altitude from each vertex in an acute triangle,
  b) Measure the lengths of the altitudes and the area of the triangle,
  c) Repeat the procedure on a right triangle, an equilateral triangle, and an obtuse triangle,
  d) State your conjectures.
- Under certain circumstances, it may be useful or necessary to provide students with more detailed instructions that lead them step by step through a construction procedure.

Drawings
Pictures are worth a thousand words. By providing drawings or withholding them, we can change the amount of information with which students begin a problem.

Tables or Charts
Tables to fill out or charts to complete help focus student attention and provide direction and guidance. Tables and charts can also serve as models for how to organize data in written formats.

Other variables that affect structure are the context in which a problem is used, the purpose for which a particular problem is chosen, and the instructional style.

To illustrate these variables, here is an example of a geometry idea, as stated in problem T 55, using that idea in three different modes: focused discovery, application, and extension. The original problem T 55 provides less direction than the focused discovery mode, but is more specific and limited in scope than the application mode example.

The examples that follow do not by any means exhaust all the possibilities. They are merely intended to demonstrate how you can use *The Supposer* to create problems that exploit the richness of a geometric concept. Start with a single idea and a willingness to explore, to take apart, to stretch, and to recombine, and you'll be able to meet a range of learning styles, needs and objectives.

The Original Problem T 55

T 55
Triangular Sections

Line segments divide any triangle into triangular sections. For example, an angle bisector divides a tdangle into two triangular sections and three medians divide any triangle into six triangular sections.

Task: To describe methods of subdividing triangles into subtriangles that have equal area.

Procedure:

- Construct any $\triangle ABC$.
- Draw line segments.
- State your conjectures.

Drawings & Data

Conjectures

What kind of segment(s) or combinations of segments and how many of them divide(s) any triangle into two sections with equal areas? ____________

Three sections with equal areas? ____________

Four sections with equal areas? ____________

Five sections with equal areas? ____________

Line segments divide any triangle into triangular sections. For example, an angle bisector divides a triangle into two triangular sections and three medians divide any triangle into six triangular sections.

What kind of segment(s) or combinations of segments and how many of them divide(s) any triangle into two sections with equal areas? What about three sections with equal areas? Four sections with equal areas? Five sections with equal areas?

The objective of this problem is to explore the properties of constructions and the subdivision of triangles. This general idea can be adapted and applied to more specific objectives and to a number of different pedagogical approaches.

The three examples on the following pages include the learning objective, the problem statement and some possible student conjectures or outcomes. You may want to compare the objectives and problem statements before you work through the conjectures.

1. Focused Discovery

The Objective:

The Supposer can be a tool for students to "invent" or "discover" definitions and theorems. Teachers who use this approach have found that the problems, especially early in the year, must point students in the proper direction and clearly define the objective. Otherwise, they run the risk that students will not discover the appropriate content. In this mode, we can rewrite problem T 55 to introduce the idea that one property of a median is that it divides the area of the triangle in two.

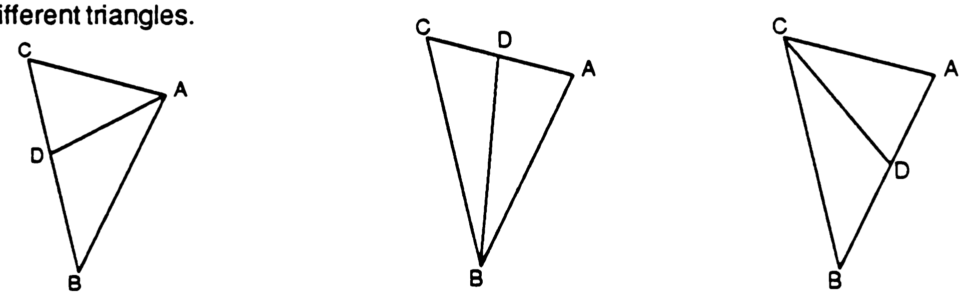
The Problem Statement:

> **Task:** To investigate one property of a median in a triangle.
>
> **Procedure:**
> - In any triangle ABC, use a median to divide the triangle into two equal areas.
> - Using medians, how many different ways can you divide the triangle into two equal areas?
> - In which types of triangles are the two triangles of equal area also congruent?
> - In which types of triangles does the drawing of the median produce right triangles? isosceles triangles? obtuse triangles?

This problem focuses on one special property of the median – that it subdivides a triangle into two equal areas. Students need to understand that this property is independent of the type of triangle and the vertex from which the median is drawn. With these ideas, students will begin to break connections in their minds between shapes and the area of shapes. They should start to think of area in terms of the length of a side (the base) and the altitude.

Possible Conjectures:

Students find three different constructions (the three medians) for dividing the area in two. It's the same method, but the three constructions carried out on the same triangle yield six different triangles.



In asking about congruence, students need to understand that if AC = AB, then the two triangles will be congruent. Therefore, this is the case in isosceles and equilateral triangles.

After students understand that the median divides the area into two equal parts, ask them to attend to the **type of shapes** and not the relationship between the two shapes created by the median. These are the kinds of results students might observe:

Right triangle:	a median drawn from the right angle creates two isosceles triangles of equal area (which are not congruent).
Isosceles triangle:	a median drawn from the angle between the two equal legs in an isosceles triangle forms two right triangles.
Obtuse triangle:	a median drawn from one of the acute angles will create two obtuse triangles of equal area.

2. Application Mode

The Objective:

The Supposer can illustrate or demonstrate concepts for a class. It can give students firsthand experience with a theorem or definition that they have learned in class or from a text. It can ask students to apply their knowledge in a straightforward problem. Here is one way to adapt the underlying idea of problem T 55 to the demonstration or application mode.

The Problem Statement:

Task: To develop procedures for using constructions to subdivide triangles into four equal parts.

Procedure:

- Draw an acute triangle.
- Find at least three different methods to subdivide any acute triangle into four equal parts.
- Record each method.
- Provide a convincing argument for each method.
- Test that each method works for all types of triangles.

In this case, we assume that students know about the properties of different elements and constructions in triangles. Here we want to focus on one property shared by different elements: subdividing area. Recording procedures requires that students be systematic in their efforts. Convincing arguments require a different kind of knowledge and understanding directly related to proof. One technique to build toward formal proof is to ask students to formulate their conjectures and convincing arguments in an "if...then..." format. Since we are not interested in a broad, expansive project, we have limited this problem to $n = 4$.

Possible Conjectures:

Here are five different methods (Can you think of others?):
(1) Using midsegments.
(2) Subdividing one leg into four equal parts.
(3) Using a combination of midsegments and medians.
(4) Using a combination of medians.
(5) Using a different combination of medians and midsegments.

The convincing arguments should build on the properties of the elements involved (*e.g.*, parallel segments, medians, midsegments).

Finally, all of the above methods work in any triangle from any vertex.

3. Extension

The Objective:

Yet another way to use *The Supposers* is as a tool to integrate and extend student knowledge. This is the objective of most projects mentioned above. Here is a project based on problem T55.

The Problem Statement:

Task: To explore and to define methods for subdividing triangles into equal parts.

Procedure:

- Draw any triangle ABC.
- Find at least two methods for subdividing the triangle into two equal parts.
- Record your methods.
- Repeat the procedure for three equal parts, four equal parts, five equal parts, and six equal parts.
- State your conjectures.

Challenges:

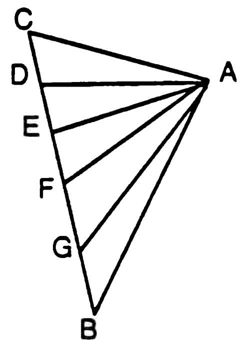
- Can you develop a method that will work when n = 3, but not when n = 5?
- There are methods that will work for any triangle, for any n. Can you find a method that is specific to a set of numbers?
- Can you suggest at least one method to subdivide a triangle into triangles with areas that are in a ratio of 3:1?

This is not a problem for novices. The scope and the absence of specific instructions would only frustrate students who lack a solid foundation in geometry and experience in inductive work. But it is an interesting and engaging project for students at the end of the year.

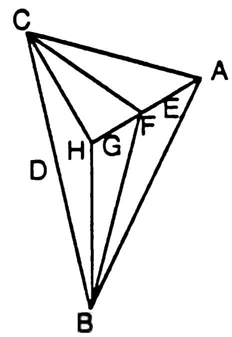
Possible Conjectures:

There are at least two methods that will work in any triangle.

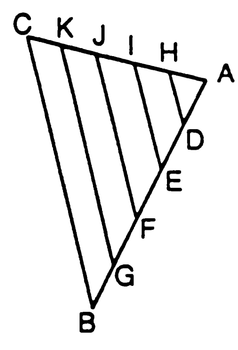
One is to subdivide any side in n parts and connect the points to the opposite vertex.



Another method is to draw a median and subdivide the median into n parts. Connect the remaining vertices to every other subdivision. If n is an odd number, then you will have to erase the last small segment on the median.

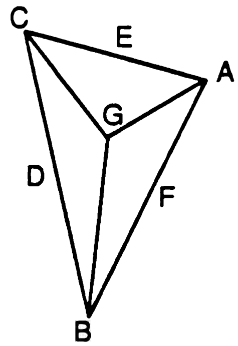


Students will probably try to build on their knowledge of parallel lines and similarity and come up with methods such as the following, which does not work:



In thinking about a method that works for n = 3, but not for n = 5, the investigation should start to focus on specific and general strategies.

Here is a method that will work for n = 3. In any triangle, draw the three medians and erase the shorter segments of the medians.

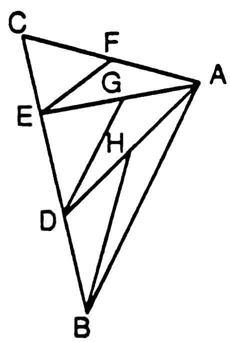


The areas of the three triangles are equal.

This will work only for n = 3. It will work for n = 6 if you do not erase the segments of the medians, but it will not work for n = 5. It leads to the following discussion of combinatorial arguments and methods that work for sets of numbers.

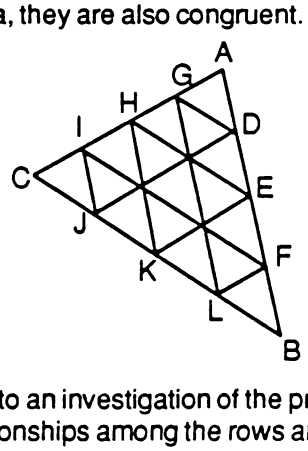
If I know how to subdivide to *k* equal areas and also how to subdivide to *l* areas, then I know how to devise a method for *k* • *l* areas.

Example: If n = 3 is known and n = 2 is known, then n = 6 is known.



If n = 4 is known and n = 9 is known, then a method for subdividing into any n that is a whole square is known.

Example: n = 16. Subdivide each side of the triangle into the square root of n parts (in this case, 4) and connect all of the subdivisions. The triangles created are not only of equal area, they are also congruent.

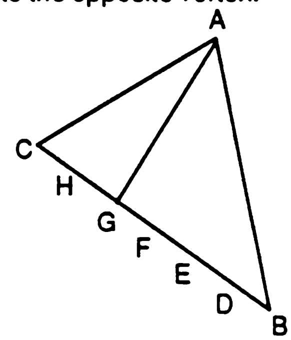


This line of inquiry could also lead to an investigation of the properties of numbers in the context of geometry (*i.e.*, the relationships among the rows and the number of triangles in the example above).

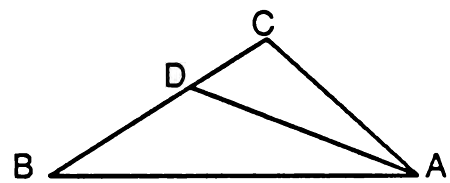
And finally, the subdivision of a triangle into triangles with ratio of $k{:}l$ area. This project gets even more interesting when you raise the possibility of subdividing a triangle into triangles which are not equal in area. Adding this wrinkle to the project will call for students to draw on and integrate more knowledge. It is also likely to engender a greater sense of discovery than the previous parts.

There are many possibilities. Here are three:

Subdivide one side of a triangle into $k + l$ parts and draw a segment from the subdivided side to the opposite vertex.

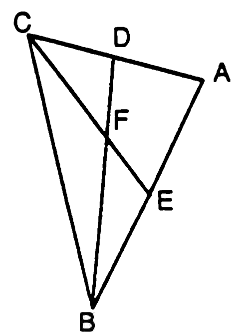


Using the Your own option on the New triangle menu, construct a triangle with sides in ratio of $k{:}l$. Draw the angle bisector of the angle included in these two sides.

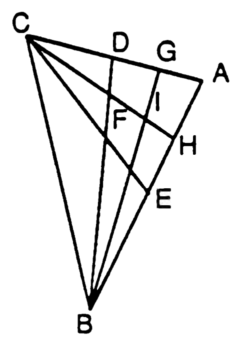


You may also wish to direct your students to investigate specific cases of ratios and cases where the subparts are not just triangles.

For example, here is a triangle in which the ratio of triangle ABC to triangle BFC is 3:1.



Here is a triangle and construction in which the ratio of triangle ABC to triangle BIC is 5:3.



Making Your Own Problem Sheets

The blank template found on the following page can help you create problem sheets for your classes. Photocopy the template, then word process, type, or write the Task and Procedure you choose. Add a drawing or a chart if it is appropriate.

You can create problem sheets by:

- modifying a problem in this book, as suggested in the previous section,
- adapting a problem from your text or creating a problem to complement the text,
- writing an original problem based on your own geometry experiences, or
- writing a problem drawn from the discoveries and conjectures your students make when working with *The Supposers.*

Be sure to begin by photocopying the blank template.

Consider sending your original problem sheets to The Geometric Supposer Society to share your ideas with other teachers. For more information on The Geometric Supposer Society, refer to the page at the back of this book.

Task:

Procedure:

Drawings & Data

Conjectures